DETERMINATION OF THE RHEOLOGICAL PROPERTIES OF LIQUIDS FROM RESPONSE FUNCTIONS OF THE FLOW STRUCTURE IN A CAPILLARY TUBE

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A method is proposed for determining the rheological properties of liquids from response functions recorded during flow of the liquids in capillary tubes, and equations are derived connecting shear stresses and velocity gradients with parameters of the response functions.

There are currently several works in which functions of the distribution of residence time (response functions) were determined for the laminar flow of viscous and nonviscous liquids in tubes [1-3].

Thus, the problem of determining response functions from known flow kinematics poses no particular difficulties.

The inverse problem of determining the velocity profile v = v(r) in tubes from the response functions was solved in [4]:

$$\left(\frac{r}{R}\right)^2=-v_{\mathrm{av}}^2\int C(v)\,\frac{dv}{v^3}+k.$$

Solution of the latter problem presupposes the possibility of finding the velocity gradient $\dot{\gamma} = \dot{\gamma}(r)$ from the response functions in a capillary tube and, in the final analysis, of determining the rheological function $\dot{\gamma} = \dot{\gamma}(r)$.

Actually, in the laminar flow of liquids in a capillary tube, the fraction of particles moving a distance r from the axis in an annular cross section is determined by the equation [1]

$$\frac{dq}{q}=\frac{2\pi r v dr}{\pi R^2 v_{\mathrm{av}}}=Cd\theta. \tag{1}$$

If the length of the capillary tube is L, then the time t = L/v, and the mean residence time $\bar{t} = L/v_{\mathrm{av}}$. Then $\theta = v_{\mathrm{av}}/v$ and

$$d\theta=-(v_{\mathrm{av}}/v^2)\,dv. \tag{2}$$

Substituting the value $d\theta$ in (1), we obtain

$$\dot{\gamma}=-\frac{2rv^3}{Cv_{\mathrm{av}}^2R^2}. \tag{3}$$

On the other hand, from Eq. (1)

$$\frac{2rvdr}{R^2v_{\mathrm{av}}}=Cd\theta, \tag{4}$$

and after integration with allowance for (2) we obtain

$$(r/R)^2=\int_{\theta_1}^{\theta} C\theta d\theta.$$

Substituting the value r/R in (3), we have

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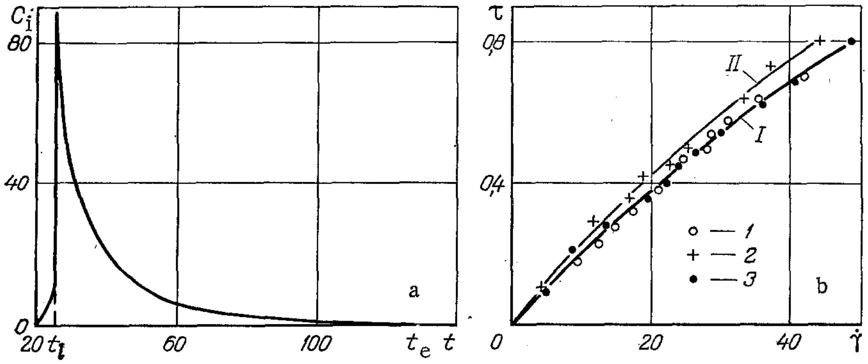

Fig. 1. Curves for determining the rheological properties of a 1.5% CMC solution in water: a) response C_i-curve obtained in the flow of this solution in a capillary tube of 1-m length and 4-mm diameter with a pressure gradient of 64 mm; b) rheological flow curves: 1) experimental points obtained after replotting the C_i curve with Eqs. (7) and (8); 2) experimental points corresponding to the dependence of the shear stresses at the wall of the tube on the mean velocity gradient (curve II); 3) experimental points corresponding to dependence of shear stresses at tube wall on velocity gradient at the wall (Weinsberger–Rabinowitsch–Mooney) (curve I). C_i, mm; t, sec; τ, N/m²; $\dot{\gamma}$ = dv/dr, sec⁻¹.

$$\dot{\gamma} = -\frac{2v_{av}}{CR\theta^3}\Big(\int\limits_{\theta_l}^{\theta} C\theta d\theta\Big)^{0.5}.$$

Converting to the dimensionless variables C and θ,

$$\dot{\gamma} = -\frac{2L\,(C_{i0}\bar{t}^2)^{0.5}}{RC_i t^3}\Big(\int\limits_{t_l}^{t} C_i t dt\Big)^{0.5}. \tag{5}$$

Since we do not know the mean residence time $\bar{t}$ or the mean concentration C_{i0}, we will use the property of the C-function [5] $\int\limits_{\theta_l}^{\infty} C\theta d\theta = 1$, from which

$$C_{i0}\bar{t}^2 = \int\limits_{t_l}^{t_e} C_i t dt. \tag{6}$$

Finally, substituting the value $C_{i0}\bar{t}^2$ from Eq. (6) in (5), we obtain the velocity gradient in the capillary tube as a function of time and the response function

$$\dot{\gamma} = -\frac{2L}{RC_i t^3}\Big(\int\limits_{t_l}^{t_e} C_i t dt\Big)^{0.5}\Big(\int\limits_{t_l}^{t} C_i t dt\Big)^{0.5}. \tag{7}$$

Since τ = Δpr/2L in the flow of any liquid in a capillary tube, then from (4) and (6)

$$\tau = \frac{\Delta pR}{2L}\Big(\int\limits_{t_l}^{t} C_i t dt \Big/ \int\limits_{t_l}^{t_e} C_i t dt\Big)^{0.5}. \tag{8}$$

Equations (7) and (8) connect the shear stresses and velocity gradients in parametric form, which is convenient for plotting a rheological flow curve.

Figure 1 shows the results of measurement of the rheological properties of a 1.5% solution of carboxymethylcellulose in water from the response C-function and Eqs. (7) and (8).

The response C-curve was determined using the well-known method employed in [6]. For this purpose, in a solution flow in a capillary tube 1 m long and 4 mm internal diameter with

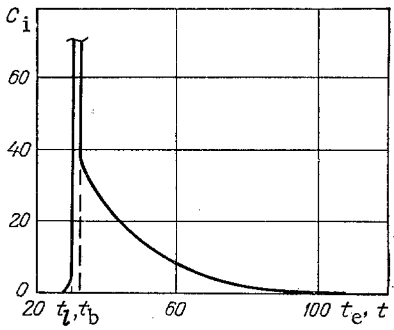

Fig. 2. Response C_i curve for liquids having a limiting shear stress.

a pressure drop corresponding to 64 mm H_2O, we introduced about 1 ml of the same CMC solution — but saturated with NaCl — in the beginning of the capillary tube to serve as an indicator. A concentration meter installed at the tube outlet automatically determined and continuously recorded the concentration of outgoing particles of the indicator in the form $C_i = C_i(t)$ (Fig. 1a). This function was replotted with Eqs. (7) and (8) into a rheological flow curve (Fig. 1b). Also shown here and in Fig. 1b are results of capillary viscometry of the same solution by the well-known Weinsberger—Rabinowitsch—Mooney method [7].

While the well-known and proposed methods of measuring the rheological properties of liquids are nearly the same in terms of accuracy, the latter is much more productive: it requires a single test at a fixed flow rate, instead of the 10-12 tests at different rates and pressure drops typical of the Weinsberger—Rebinowitsch—Mooney method.

Also, the inexact operation of numerical differentiation in the well-known method of capillary viscometry is replaced by the more accurate operation of numerical integration by Eqs. (7) and (8). However, the proposed method of measuring rheological properties from the response C-curve in a capillary tube applies to liquids, where molecular diffusion of the indicator is very slight in recording residence-time distribution functions [8, 9]. For gases, where molecular diffusion is comparable to convective mass transfer, the response function characterizes not only the velocity profile in the capillary tube, but also the distribution of indicator in the longitudinal and transverse directions due to concentration gradients and diffusion. Thus, replotting the response C-function of a gas flow in a capillary tube into a rheological flow curve with Eqs. (7) and (8) could lead to substantial errors.

In determining the rheological properties of a large class of non-Newtonian fluids having a limiting shear stress τ_0 from the response function, difficulties may arise in connection with division of the flow as a whole in the tube into zones of nongradient and gradient flow [10].

Liquid particles located in the nongradient zone leave the tube nearly simultaneously at time intervals (t_l—t_b), where $t_b \to t_l$. This is reflected on the response curve (Fig. 2) in the form of a concentration peak in the time interval (t_l—t_b). The intervals for Eqs. (7) and (8) characterizing the moments of the areas under the response C curve cannot be exactly determined because of the imprecise recording (due to recorder inertia) of the concentration of the indicator in the vicinity of the peak. The following procedure can be used in this case: the mean residence time $\bar{t}$, in accordance with its physical significance, is determined from the formula $\bar{t} = \pi R^2 L/q$, while the mean residence time of the particles in the gradient zone, as the coordinate of the geometric center of gravity of the area under the response curve in the interval (t_b—t_e), is determined from the equation

$$\bar{t}_g = \int_{t_b}^{t_e} C_i t\,dt \Big/ \int_{t_b}^{t_e} C_i\,dt.$$

From the well-known formula for the geometric center of gravity of a complex figure

$$\Big(\int_{t_l}^{t_e} C_i\,dt\Big)\bar{t} = \Big(\int_{t_l}^{t_e} C_i\,dt - \int_{t_b}^{t_e} C_i\,dt\Big)t_l + \Big(\int_{t_b}^{t_e} C_i\,dt\Big)\bar{t}_g \tag{9}$$

we find the expression for determining the total area under the response C curve.

Proceeding on the basis of the equation describing the fraction of particles in the nongradient zone

$$\frac{dq}{q}=\frac{\int_{t_l}^{t_e} C_i\,dt-\int_{t_b}^{t_e} C_i\,dt}{\int_{t_l}^{t_e} C_i\,dt}=\frac{\bar{t}_g-\bar{t}}{\bar{t}_g-\bar{t}_l}$$

and relations connecting the fraction of particles in this zone with the radius of the same zone

$$\frac{dq}{q}=\frac{\pi (r^*)^2 v_n}{\pi R^2 v_{av}}=\left(\frac{r^*}{R}\right)^2\frac{\bar{t}}{t_l},$$

we obtain an equation for determining the radius of the nongradient zone

$$\left(\frac{r^*}{R}\right)=\left[\left(\frac{\bar{t}_g-\bar{t}}{\bar{t}_g-t_l}\right)\frac{t_l}{\bar{t}}\right]^{0.5}$$

and the limiting shear stress

$$\tau_0=\frac{\Delta pR}{2L}\left(\frac{r^*}{R}\right).$$

Taking Eq. (9) into account, we write the total initial first-order moment in the form

$$\int_{t_i}^{t_e} C_i t\,dt=\left[\frac{\bar{t}(\bar{t}_g-t_l)}{\bar{t}_g(\bar{t}-t_l)}\right]\int_{t_b}^{t_e} C_i t\,dt.$$

Similarly, we express the running initial first-order moment as follows

$$\int_{t_l}^{t} C_i t\,dt=\left[\frac{t_l(\bar{t}_g-\bar{t})}{\bar{t}_g(\bar{t}-t_l)}\right]\int_{t_b}^{t_e} C_i t\,dt+\int_{t_b}^{t} C_i t\,dt.$$

Substituting the values of total and running first-order moments from the last two equations into (7) and (8), we obtain a relation for determining the velocity gradient and shear stresses in the gradient zone.

NOTATION

C, dimensionless concentration of indicator, differential response function; C_i, C_{io}, values proportional respectively to the concentration and mean concentration of indicator on the machine response curve, mm; k, integration constant; L, length of capillary tube, m; q, flow rate of liquid, m^3/sec; R, radius of capillary tube, m; r, radius of layer of liquid in capillary tube, m; $\bar{t}$, mean residence time, sec; v, velocity, m/sec; $\dot{\gamma}$, velocity gradients, sec^{-1}; Δp, pressure gradient in the capillary tube, N/m^2; θ, dimensionless time; τ, shear stress, N/m^2; τ_0, limiting shear stress, N/m^2. Indices: g, gradient zone; l, lag; e, end; b, beginning; n, nongradient zone; av, mean value.

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ONE APPROACH TO CALCULATING A TURBULENT BOUNDARY LAYER ON A SURFACE WITH A COMPLIANT COATING

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A method is proposed for calculating a turbulent boundary layer on a surface with a viscoelastic coating. The method is based on the introduction of the van Driest damping function to account for the effect of the coating on the boundary layer.

It is now considered proven that the application of a layer of viscoelastic (elastic, compliant) material to the surface of a body moving in a liquid or gas may lead to a 50-60% reduction in the drag associated with the body. This has been confirmed by several experiments with different types of coatings (we may recommend the survey [1], which contains an extensive bibliography). At the same time, there are studies in which the investigators not only failed to find a reduction in drag, but in fact observed the reverse effect.

Theoretical study of a turbulent boundary layer on such surfaces is complicated not only by transient boundary conditions, but also by a lack of detailed knowledge of the dynamics of viscoelastic materials. Only in the most recent works [2, 3] have attempts been made to take a combined approach to this problem.

Friction on a solid surface is limited by the interaction of turbulent and viscous transfer near the surface. One effective approach to accounting for the interaction of viscous and turbulent transfer in a turbulent boundary layer close to a solid surface is the introduction of so-called damping functions, reflecting the dynamics of pulsations in a viscous fluid. Such functions have been obtained by different methods by Loitsyanskii, Vulis, and van Driest [4] for a boundary layer on a flat plate. For example, van Driest proposed a structural form of damping function for the length of the displacement path l in a plane boundary layer on the basis of an analysis of harmonic oscillations of an infinite flat plate in an unbounded incompressible viscous fluid (the Stokes problem) decaying as they penetrate into the fluid according to the law $\exp[-y(\omega/\nu)^{1/2}]$, where ω is the frequency of the oscillations; ν is the kinematic viscosity coefficient of the liquid:

$$l^+ = \varkappa y^+ F(y^+),\ F(y^+) = 1 - \exp(-y^+/a), \tag{1}$$

where $y^+ = yu_*/\nu$; $\varkappa = 0.4$; $a = 26$ are universal empirical constants.

Further, several authors used van Driest's idea as a basis for constructing damping functions for certain more complex flows (turbulent boundary layer on porous flat [5] and cylindrical [6] surfaces).

The attractiveness of van Driest's idea derives first of all from the fact that well-known solutions of the Navier—Stokes equations are used (in one form or another) to construct a structural form of damping function for a given complex turbulent flow. In such a situation, it would be of interest to construct a damping function of the van Driest type which would convey information on the properties of the viscoelastic coating mentioned earlier.

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